UNIVERSITY OF ILORIN

THE TWO HUNDRED AND SEVENTH (207TH) INAUGURAL LECTURE

MAKING THE BEST USE OF WHAT YOU HAVE: THE MATHEMATICAL APPROACH

BY

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THURSDAY, 4TH NOVEMBER, 2021

This 207th Inaugural Lecture was delivered under the Chairmanship of:

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4th November, 2021

ISBN: 978-978-57721-3-5

Published by

The Library and Publications Committee
University of Ilorin, Ilorin, Nigeria

Printed by
Unilorin Press
Ilorin. Nigeria

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Friends and Relations,
Distinguished Invited Guests,
Great Unilorites,
Members of my family – nuclear and extended,
Distinguished Ladies and Gentlemen.

Preamble

I begin this Inaugural Lecture in the name of God the Father, God the Son, and God the Holy Ghost, acknowledging the love, goodness and kindness of the triune God from Whom "Every good gift and every perfect gift" (James 1:17) comes. It is "Not of him that willeth, nor of him that runneth, but of God that sheweth mercy" (Romans 9:16). It is God's mercy that saw me through till date, and His mercy will 'follow me all the days of my life', Amen.

It is difficult for me to fathom how someone who was a consistent truant in early primary can befriend school to the extent of being counted among the scholars; it is the unmerited grace of the Almighty. “This is the Lord’s doing; it is marvelous …” (Psalm 118:23).

‘Born and bread’ in the city of Lagos, I would leave home for school, but in between were so called friends who diverted my way to the jungles to scavenge for plumy mango and citrus fruits, and other things. Unknown to me then, my future was gradually being bargained for destruction. But I thank God that it was not long until my hanky-panky was uncovered. My parents decided to send me to their home town, far away from Lagos, to continue my education, and that was how I was rescued from an early-life disaster!

Thank God, the rest of my journey in acquiring education was a smooth course. From the primary to the secondary, God continuously guided my pathways with His “unseen hand.” I cannot but thank and thank God for bringing me into a personal relationship with Him through the ‘born-again’ experience. I acknowledge Him as the source of my successes in life.

The University of Ilorin gave me undergraduate admission to study Education and Mathematics for which I still remain very grateful because that was my gateway to academic success. At the end of my study, I was awarded a Bachelor of Science in Education and Mathematics with First Class honours.

Having drank from the nourishing fountain of knowledge issuing from the ‘Better by Far’ University once, I came again and again, until I eventually ‘pitched my tent’ at my alma matter, where I have been for more than thirty years. I give glory and honour to God Almighty
that the ‘run-away’ from school of those days has found a befitting citadel of learning to abide.

Introduction

This Inaugural Lecture titled, "**Making the Best Use of What You Have: The Mathematical Approach**" is the two hundred and seventh in the series at this university, and the sixth from the Department of Mathematics, Faculty of Physical Sciences. The first Inaugural Lecture from the department was presented by my mentor, late Professor Matthew Adeyanju XIbiejugba, on Thursday 24th May, 1990 with the title: "Living in the Best of the Possible Worlds".

I equally acknowledge and appreciate the efforts of the remaining four colleagues, Professors T.O. Opoola, R.B. Adeniyi, J.A. Gbadeyan and K. Rauf from our department, whose Inaugural Lectures preceded mine for 'keeping the mast flying high.'

From the title of the first afore-mentioned Inaugural Lecture, I believe you will notice some parallel in the qualifier 'best' and the contexts 'living' and 'making'. The similarity is because both of us specialize in the same area of mathematical studies.

Mr. Vice-Chancellor Sir, as I set out in this presentation, permit me to clarify that the topic of this lecture: **Making the Best Use of What You Have: The Mathematical Approach** is situated in a field of Mathematics known as Mathematical Optimization. It is my intention to use this auspicious occasion to present technical and working definitions of optimization upon which platform I will address on optimization in nature before considering things that we have and how to apply Mathematics to make best use of them. Presentation of my humble contributions to this important field of endeavor will be the next, and then follow the conclusion, recommendations and acknowledgements.

Optimization Defined

The verb 'optimize', according to the web-based Oxford Dictionary, is "to make the best or most effective use of a situation or resource." Synonyms of the word include improve, enhance, elevate and raise. Thus technically, optimization is the act of making the best or most effective use of a situation or resource.

The word ‘best’ may mean one of two things depending on the setting. For example, a trader would naturally want to maximize his profit and minimize the cost of his wares in order to sustain his business. Another illustration is that of a vehicle owner, who will prefer to follow a longer, or even the longest route, to his destination in a bid to preserve the lifespan of his automobile.

As such, optimization is an exercise of maximizing or minimizing an outcome for a definite purpose. That means the goal of any optimization effort is to maximize the desired benefit or minimize the effort required in carrying out the activity. Thus, a working definition for optimization is: an enterprise of maximizing or minimizing for a definite purpose.

It is necessary to note that the exercise of selecting the ‘best’ often does not result in a dichotomy, but may present an array of alternatives from which an option has to be selected according to some criterion. For instance, in trying to access the peak of a mountain, one may need to settle for another top which is not the highest, depending on the landscape of the surroundings, see Figure 1.

Figure 1: Atop a Mountain Peak
Source: Google.com

The situation that necessitates a search for the 'best' alternative is referred to as an optimization problem, and the best alternative is called the solution. Two important issues can be seen to emerge in the context of optimizing, namely, decision making, and meeting some standard(s). A third element has to do with conforming to the setting where the 'best' is sought.

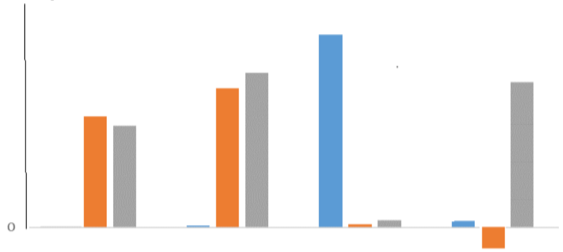
Optimization in Nature

The Vice-Chancellor Sir, Ladies and Gentlemen, there are several occurrences in nature that are concerned with minimizing or maximizing certain quantities. As humans, the application of optimization principles are exhibited in our everyday life by the way we work, the choice of line we queue in at the ATM facility, or the decision we take on the welfare of our loved ones.

The way the ever-busy swarm of bees build their home is typical of optimization in nature. The hexagonal shape of the honey bee cells is a geometry that demonstrates magnificence of optimization, see Figure 2. Varro (37 B.C.) opined that the patterns of the bee hive in comparison to other polygons possess the maximum ratio of surface to perimeter in order that the bees may attain the best-use of available materials. The empirical results of Hepburn (1986) concerning beeswax production later justified the assertion of Darwin (1859) that the bees adopted the geometrical construction observed, in a bid to waste the least quantity of honey.

Figure 2: Hexagonal geometry of the bee cells
Source: Google.com

Planetary Science affirms that of the four terrestrial or inner planets - Mercury, Venus, Earth and Mars – the Earth is the only place where life can be found. The reason for this is because God, the Creator of the entire universe, applied the optimization act to make the Earth the planet that is most abundant in water, whose atmosphere is the freest from harmful gases and solids, and which has the optimal temperature range that makes it the most habitable for living things, being not too hot and not too cold. Availability of the three resources are illustrated in Figure 3.



	Mercury	Venus	Earth	Mars
Water Concentartion	0.00000 km^3	0.000004 km^3	1386000000 km^3	562000 km^3
Temperature	430℃	461.85℃	14.9℃	-63°C
Harmful Gases Level	75%	98%	5%	95%

Figure 3: Charts comparing basic resources at habitable planets

My last illustration of maximization and minimization principle is the fact that our Inaugural Lectures are developed to the full or maximum capability, but time constraint limits the delivery to a minimum effort.

What we have to Make Best Use of

Mr. Vice-Chancellor Sir, it is pertinent to ask at this stage: What do we have that we are to make best use of? My answer is: many things! As a creative act of God, the individual is given life in all its ramifications – health, talents, possessions and a future - to live to the greatest of his potentials. As a society – family, institution or nation – we are gifted with abundant resources and possessions, which we are expected to use profitably.

The things we have that we are to make best use of are varied, but they can be summed up as our resources. According to Wikipedia, resources are all possessions which assist us to satisfy our necessities. The Collins Dictionary went a bit further to categorize resources into individual and collective entities.

As individuals, each of us is endowed with two classes of essential resources, namely supernatural resources that are very delicate, and external resources that are valuable. Our supernatural resources include our life, health and future. The external ones consist of money and other valuable properties. The main objective of being given these possessions is to optimize their use. The resources of a corporate body like a company, include financial, human, material and intellectual resources. In contrast to the afore-mentioned entities, a nation has in addition ample natural resources such as coal, oil, natural gas, metals and other raw materials, Berry (2004).

Life

Scientifically, life is “Any system capable of performing functions such as eating, metabolizing, excreting, breathing, moving, growing, reproducing, and responding to external stimuli” (Margulis, Sagan and Sagan, 2020).

We often hear the expressions: ‘Life is precious’, ‘Life is sacred’ and ‘Life is short.’ These are facts of reality. The preciousness of life is borne out of the value of life, which is the rarest resource.

In order to therefore make optimal use of the short, ephemeral and mortal life, we need to do a few things. One of these is to live and let others live. This emphasizes the importance of harmonious living in the human society. Peaceful coexistence is important to a successful life, Rani (2019). Another strategy is to imbibe the attitude of hard work, utilizing all our talents and gifts profitably well.

As a creation of a holy, pure, kind and merciful God, our life must be spent to please Him. According to Cranft (2018), we must reflect God’s characters in order to actually live a truly fulfilled life.

Money

Another important possession is money. With money, we buy goods and services. Money is a measure of quantity of transaction in a business. There are three primary functions of money; it is a medium of exchange of goods and services, a unit of measurement for accounting purpose, and a valuable treasure that can be stored for preservation. Although money is not everything, nonetheless it is very important, for it can aid us to easily achieve our goals, Ganeshan (2010).

Other Resources

The resources available to a nation are of two types, the natural and man-made resources. Natural resources can be identified with renewable and nonrenewable sources. Man-made resources include machinery, finished goods and industrial products which man uses natural things to improvise in order to provide utility and value to our lives.

One major challenge to conservation of natural resources is the rate at which the resources are being depleted. This issue has been the major focus of government of various nations including the United Nations. As a result, the world body has produced a document on Sustainable Development tagged UN Agenda 21. Section Two of the agenda outlines the necessary steps that nations are to follow to conserve and manage their natural resources.

To mitigate the depletion of natural resources and its impact on sustainable development calls for the application of principles and strategies of optimization.

Techniques for Resource Management

Resource management has been defined as the process of administering resources in the best possible way. The skills involved in the process are planning, scheduling, allocating, forecasting and utilization. There are a number of techniques applied in the bid to achieve efficiency and effectiveness in managing resources. The following practices have been identified for optimal usage of resources:

Process improvement: Process improvement is the task of identification and analysis of a process that ensures improvement in resource utilization. Continuous improvement in a procedure usually perfects it and eventually leads to an optimal system.

Waste elimination: Waste elimination as business concept, is one of the most effective means to improve on profitability. The idea is built on the removal of any activity that does not add value to the procedure for resource utilization. Therein, efforts are to be made to cut down on waste in a bid to improve resource availability and utilization.

Intuition: Intuition is defined as the ability to understand something without the need for conscious or analytical reasoning. It is a powerful force that helps the mind to make better decisions. Since optimization is basically a decision science, intuition finds relevance in the resource management. The dominant role of intuition provides the foundation for logical reasoning.

Mathematical Optimization

Mr. Vice-Chancellor Sir, distinguished ladies and gentlemen, the usefulness of ideas emanating from studies in resource management is not in doubt, but there is a need to utilize techniques and strategies that are built on sound theoretical foundations. One of such fields of study is Mathematical Optimization.

As most real-world applications are concerned with minimizing or maximizing some quantity so as to enhance some result, Mathematical Optimization as a branch of applied mathematics, finds application in different areas of human endeavour. Typical applications from different fields are listed in the table below.

Table 1: Applications of Mathematical Optimization

OPTIMIZATION			
Pure & Applied Sciences	**Engineering**	**Management & Social Sciences**	**Comm. & Info. Sciences**
Optimization of Physical Quantities	Automatic Control Systems	Decision Science	Machine Learning
Power Dessipation	Signal Processing	Data Management	Artificial Intelligence
Energy Minimization	Communication & Networks	Security Management	Data Visualisation
Optimal Experimental Design	Product & Shape Design	Mix Products and Services	Neural Networks
Metabolic Engineering	Truss Topology Design	Supply Chain Management	Algorithm Efficiency
Synthetic Biology	Electronic Circuit design	Product Costing	Management
Biochemical Reaction Networks	Data Analysis & Modelling	Supply Chain Management	Games and animations
Bioprocessing	Financial Engineering	Freight Transportation & Delivery Systems	Development Assemblage
Molecular Potential Energy	Transportation Networks	Schedulling	Runtime Analysis
DNA-Based Steganography	Production Planning	Regional & Urban Design	Compiler Tracking
Dental Image Registration	Design & Data Fitting		Computer Aided-
Image Resolution & Reconstruction	Hydraulic & Piping Engineering		Diagnosis
CT & MRI Scans			
Tumor Analysis & Treatment			
Radiation Therapy			

Mathematical Model

Finding the optimal solution commonly involves constructing a mathematical model to describe such problems. A model may be defined as a simplified representation of certain features of a real object, situation or process. An example is a toy car. A mathematical model, therefore, is a model created using mathematical concepts such as functions, equations and expressions.

Mathematical modelling is thus, the act of building models for a specific purpose. In the course of this lecture, the use of mathematical modelling to solve problems will be demonstrated. But to be able to apply mathematical modelling effectively, there must be a procedure or methodology to follow. Below is a typical methodology for mathematical modelling.

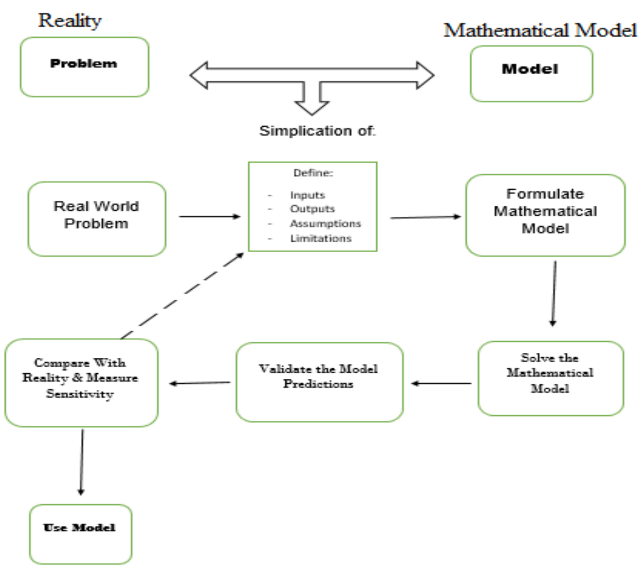


Figure 4: Methodology of mathematical modelling
Source: Google.com

Optimization Problem

Mathematically, we can symbolize an optimization problem as

Optimize $z = f(x_1, x_2, \dots, x_n)$ … (1)

subject to the constraints

$g_j(x_1, x_2, \dots, x_n) \le \text{or} \ge 0, \mathrm{j} = 1, 2, \dots, \mathrm{m}$ … (2)

and

$h_k(x_1, x_2, \dots, x_n) = 0, k = 1, 2, \dots, p$ … (3)

In the first equation, $f(x_1, x_2, \dots, x_n)$ is the objective function that is to be optimized, which means to either maximize or minimize the value of the objective function. Examples of the objective may be the expected return on an investment, the production costs or profits of a company, the number of targets $x_1, x_2, \dots, x_n$ are the variables in the optimization problem (1) – (3).

The variables are the unknown quantities to be determined so as to optimize the objective function. In addition, because the variables are meant to be used to carry out some actions, in practical situations, they are termed decision variables.

The second and third equations are expressions for inequality and equality constraints respectively. These could be the restrictions placed on the values that the variables can be assigned.

The programming problem composed of the objective only is said to be unconstrained, while those with (1) and (2), (1) and (3), and (1) – (3) respectively are classified as inequality constrained, equality constrained, or simply said to be constrained problems.

Solution to Programming Problem

The process of solving mathematical optimization entails first identifying the

$x = (x_1, x_2, \ldots, x_n)^T$ that satisfy the constraints before evaluating the objective function. While trying to ensure constraints satisfaction, there may be many $x = (x_1, x_2, \ldots, x_n)^T$ that will qualify. These are referred to as feasible solutions. But only one of the feasible solutions can optimize the objective function. That vector is pinpointed as $x^* = (x_1{}^*, x_2{}^*, \ldots, x_n{}^*)^T$ and christened the **optimal solution** to the optimization problem.

Types of solution

Mr. Vice-Chancellor Sir, please permit me to recall that a mathematical programming problem can be stated in two forms depending on whether the objective is to be minimized or maximized. These are depicted as **relative** (local) **minimum** or **maximum, global** (absolute) **minimum** or **maximum** solution. The local and global optimal solutions are illustrated in

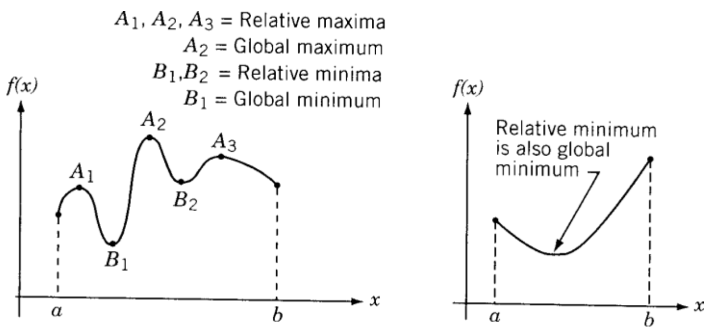
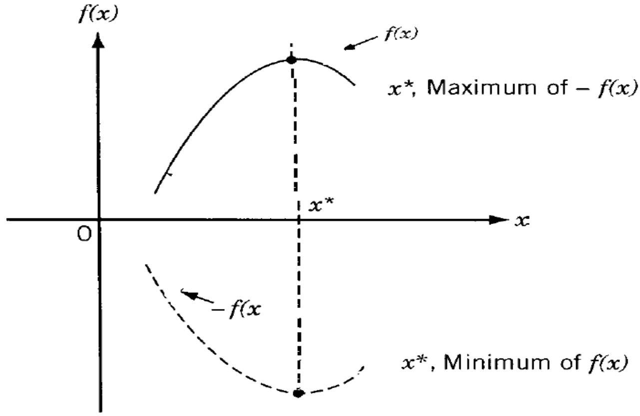


Figure 5. The two properties of the objective function can be seen from Figures 6 and 7.

Figure 5: Local and global minima and maxima

Source: Rao (2009)



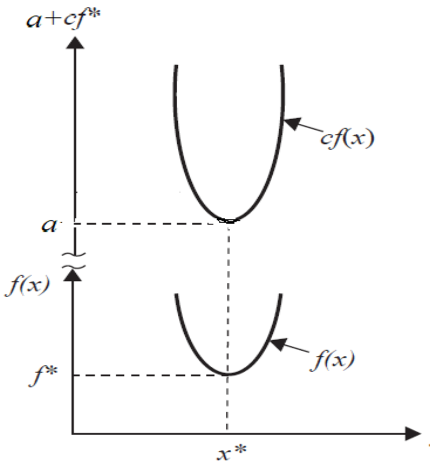


Figure 6: Maximum of f (x) is minimum of −f (x)

Figure 7: Optimum solution of a + cf(x) same as for f(x)

My Contributions to Knowledge

The Vice-Chancellor Sir, permit me to give a short background on my research activities. My research focus is in the broad area of Applied Mathematics, specifically, in Mathematical Optimization and Numerical Analysis as well as Applicable Mathematics. My efforts in Mathematical Optimization were geared towards theoretical considerations and practical applications. An aspect concerns the extension of efficient one-dimensional search methods, and formation of hybrids of such methods for multivariable optimization.

I begin with my contributions to research in Numerical Analysis, for it was the area where my M.Sc. and Ph. D. supervisor, late Distinguished Professor M.A. Ibiejugba (of blessed memory) and his academic pal, Professor P. Onumanyi, inducted me into.

Contributions to Numerical Analysis

My research efforts in numerical analysis touched on construction of approximating tools for mathematical physics problems, exploring the algebraic properties of the Gauss-Seidel iteration matrix for the solution of large-scale systems of linear equations, and designing algorithms for equitable partitioning.

Construction of Approximation Tools

In Ibiejugba, Onumanyi and **Bamigbola** (1987) and **Bamigbola**, Ibiejugba and Onumanyi (1988), linear, quadratic and cubic approximating tools were constructed based on the location of Xthe zeroes of the Chebyshev polynomials of the first kind. The constructions were validated for their governing formulae for any number of subdomains using the Principle of Mathematical Induction, and their computational efficiency was demonstrated by applying them to solve some mathematical physics problem.

Both papers were published in reputable journals. It is worthwhile to mention that the attraction for the publication of the second paper in the International Journal for Numerical Methods in Engineering, volume 26, pages 327-313, one of the topmost international outlets in the field, and published by the Taylor and Francis, was as a result of the demonstration of high-level efficiency of our computing tools for a very difficult mathematical problem where a host of other similar schemes failed. The constructions were later confirmed in Ibiejugba et al.

(1999) to yield the optimal discreet Galerkin method, and has appeared in another highly rated off-shore
journal - International Journal of Computer Mathematics, volume 72 number 1, pages 115-129.

Applying the tools in the resolution of linear ordinary differential equations (ODEs) with variable coefficients, the numerical results obtained by **Bamigbola** and Ibiejugba (1992a) showed that the approximate solution is of order (h^{2p+1}) where h and p denote the length of the subdomain and degree of the constructed tool. Also, the possibility of attaining the exact solution was demonstrated. The case of nonlinear ODEs was analyzed and demonstrated to yield accurate results even for tools of first degree, see **Bamigbola** and Ibiejugba (1992b).

Bamigbola and Adeshina (1993) demonstrated the versatility of the basis functions in the product approximation to produce accurate solutions to two-point nonlinear boundary value problems in the ODE. In addition, the product approximation scheme requires much less computational effort compared to some standard numerical method of solution.

In demonstration of the robustness of the Chebyshev polynomials, **Bamigbola**, Ibiejugba and Evans (1996, 1997) presented analyses for linear and nonlinear problems via the finite element method with the constructed tools. Advantages of these effort include the demonstration of superiority of equal interval elements over unequal ones and suitability for computer automation.

The notion among contemporary numerical analysts is that any set of basis functions can be employed to produce accurate solution to mathematical problems. As such a comparative study of the performances of three prominent set of basis functions was undertaken. The result of the experiment reported in the sole authored publication, **Bamigbola** (1996a), on pages 253-364 of a 1996 edition of the *ABACUS* volume 24 no. (2) indicated only one of the basis functions as consistently yielding accurate solutions to the same test problems at the shortest execution time. Thus, the study proved that the choice of basis functions used in the expansion method is essential.

Building on the result of the above empirical study, **Bamigbola** and Adebiyi (1997) generalized the set of basis functions to satisfy some smoothness and differentiability conditions. And making use of a computing algorithm implemented in PASCAL, the generation of the new basis was automated. With the aid of a set of numerical examples, the versatility of the newly developed tool was demonstrated.

Aliu, **Bamigbola** and Ali, M. (2018) adapted the one-dimensional approximation tool for the finite element analysis of higher-dimensional problems. The fundamental result obtained was:

Theorem:

Suppose the set of one-dimensional basis $\phi_1(x), \phi_2(x), \ldots, \phi_N(x)$ defined over the domain D are complete with local support then, the set of m-product functions $\prod_{j=1}^{m} c_j \varphi_j(x)$, where $m \leq N$ constitutes a basis for the product space D^m, which is also complete with local support and of cardinality N^m.

The result demonstrates the adaptability and ease with which the approximating tools were constructed and applied for solving higher dimensional problems. This is a major accomplishment in the research.

Bamigbola (1996b) also developed a hybrid technique, composed of the versatile finite element method with Z-transform, for the solution of the class of first order initial-value problems in the form of variable coefficient non-homogenous differential equations defined on the semi-infinite domain. Thus, blending tools in numerical analysis and linear algebra for the resolution of real-life problems. From the properties of the transform, the existence of a reasonable unique closed form solution was assured.

Exploration and exploitation of features of Gauss-Seidel iteration

My interest in the next aspect of numerical analysis was due to a curiosity aroused while teaching an undergraduate course on iterative solution of systems of linear equations. Two stationary iterative methods – Jacobi and Gauss-Seidel - were to be treated, but I observed that the convergence criterion for the former was always made use of in discussing the convergence of the Gauss-Seidel iteration. the convergence criterion of only the former was listed. When an extensive search into available literature at my disposal could not produce the criterion, the task was included as part of my personal research.

With the support of my research assistants, mainly students, the convergence for Gauss-Seidel iteration has been discovered and presented at an international forum. A preliminary report by **Bamigbola** and Ibrahim (2014) was published in the International Journal of Mathematical, Computational, Physical and Quantum Engineering.

A solution scheme was devised, in **Bamigbola** et al. (2020), by making use of the new convergence criterion, and tested on a set of standard systems of linear systems. The scheme yielded very accurate solutions, and outperforms five other methods on the basis of norm of residual of the solutions.

Bamigbola and Kitgakka (2021) exploited the discovered Gauss-Seidel's convergence criterion for solving singular linear systems of algebraic equations. The result yielded an enhanced semi-iteration and obtained the exact solution when tested on a set of singular linear systems.

I am glad to inform this audience that 2 PhD theses have been successfully awarded based on the research work, and efforts are ongoing to further exploit the results so far obtained.

Research in Mathematical Optimization

Dear Vice- Chancellor Sir, Mathematical Optimization is the mainstay of my academic training and research efforts. My research publications in this field are spread across nine (9) areas.

Constrained and unconstrained multivariable optimization

Many experts in nonlinear optimization believe that second derivative methods are more reliable because they usually require significantly fewer number of iterations and function evaluations than methods which use function or gradient information. This explains the extreme popularity of Newton and quasi-Newton methods in the field of unconstrained nonlinear programming.

However, one thing that has impeded numerical progress in nonlinear programming is lack of a convenient method for representing and coding nonlinear functions of several variables, its gradient vector and Hessian at a point that would make computer solutions easily available. But this feat was accomplished in a paper co-authored by Ibiejugba, Adewale and **Bamigbola** (1991), and published in second volume of the *Afrika Matematika*, pages 19–36. The well-known quasi-Newton method and the ingenuity of computer programming technique were wielded to carry out the direct inversion of the associated nonsingular Hessian matrices of a class of factorable functions to obtain numerical results which compared favourably with existing known results.

The classical of one-dimensional search method which obtains the optimum of a given univariate function requires several function evaluations. In order to have fewer number of evaluations, the search method was utilized to obtain the desired method. The restricted and unrestricted cases were cited as part of my research endeavor.

Ibiejugba, Agyingi and **Bamigbola** (1999) reported on the development of a multivariable Fibonacci search method using the notion of hyperplanes. Results obtained are that the number of function evaluations required is $(n - 1)m + 1$ compared to Krolak´s extension of the same restricted search given by n^m, where m is the number of evaluations along any single dimension and n, the dimension. A problem of 30 variables was executed by the new technique in about 47seconds while it took 1 hour 20 minutes to run a function of 5 variables using Krolak´s extension. The new method compared favourably with popular direct search methods of Hooke & Jeeves, and Nelder & Mead. This important result was published in the maiden edition of the International Journal of Mathematical Sciences.

Following the same approach, **Bamigbola** and F.B. Agusto (2004), published in the International Journal of Computer Mathematics, evolved a search method built on the classical Coggin´s method. The original method is a combination of the unrestricted one-dimensional search and the quadratic interpolation techniques. To cope with multivariable optimization, a search parameter was introduced to explore the descent direction $-\nabla f(x)$ thereby accelerating the rate of convergence. The solution was obtained as a bilinear form involving the square Vandermonde determinant. Computational experience in the extension highlighted the impact of the exploration of the search direction on the efficiency of the resulting method. The computational checks using standard problems showed that the multivariable method is no less inferior to other known methods.

The search technique can also be used to handle constrained problems, which are known to be more difficult computationally than their unconstrained counterpart. Ibiejugba, **Bamigbola** and Oruh (1995a) devised an ingeneous approach to solve constrained optimization problems. The strategy involved the incorporation of the pattern search of Hookes and Jeeves into the barrier function method and it resulted in a search scheme that was reliable, robust and which generally yielded the optimum solution. The paper appeared in the eighth volume of the Nigerian Journal of Mathematics and Applications.

The same trio of Ibiejugba, **Bamigbola** and Oruh (1995b) observed that the choice of the initial value R_1 and the rate at which R_k tends to 0 in the sequence $\{R_k\}$ associated with the barrier function method can seriously affect the computational effort required to attain the optimal solution x^*. An approach, which locates the optimum value of the parameter

R_k depending only on the point x_k of the constraints, was developed via differential calculus. Apart from removing the arbitrariness in the choice of the parameter, the technique also accelerated the convergence of the method, and thus, made the resulting modified barrier function method perform better than the existing classical counterpart.

Another method for solving the class unconstrained optimization problems is the trust region method, which is currently one of the best available methods for global optimization. The method is such that at iteration k, a model will be constructed to approximate the objective function $f(x)$ in a neighbourhood $\bar{B}(x_k,\Delta_k) = \{x \in \Re^n : \mathrm{k}x - x_k\mathrm{k} \le \Delta_k\}$, which is the trust region. The traditional trust region method depends on the choice of the trust region radius, a set of other parameters and uses a quadratic model for approximating the objective function. Since most objective functions are nonlinear and in particular non-quadratic, replacing the quadratic model by a higher order one is expected to improve the accuracy. In pursuit of this goal, Oruh and **Bamigbola** (2009a) developed a high order trust region method via the Taylor series expansion with an explicit updating formula obtained for the radius. The use of Taylor's result helped preserve the shape of the higher order model as well as the global convergence property, and thus, to obtain the exact solution in most of the numerical test problems.

Furthermore, as reported in the literature, the choice of parameters for the trust region radius was more or less heuristic. In order to remove the dependency, Oruh, **Bamigbola** and Adewale (2009b) solved the trust region subproblem with the application of a modified form of the standard conjugate gradient method. The resulting trust region radius uses the current iterative information and preserves the strong convergence property of the trust region method. Numerical results obtained for some selected computational problems gave indication that the iteration dependent trust region criterion is efficient,

To further improve on the accuracy of the trust region method, Oruh and **Bamigbola** (2013) developed a new dogleg method by replacing an heuristically determined parameter in the method in the implementation for nonlinear optimization. Analytically, this new approach preserved the strong convergence property of the trust region method. Computational results, on benchmark problems from the CUTE collection, showed that the new method was effective, accurate and converges in fewer iterations than with the conventional approach.

Conjugate gradient methods

Optimization problems naturally arise in a wide variety of applications in human endeavours. With the pervasive innovation in science and technology, there has been an astronomical increase not only in the sheer size of the evolving problems, but also in terms of their complexity. As a result of the complexity involved and the speed at which solutions are required, methods which yield more accurate solutions at faster speed and utilizing fewer resources are sought after. In this connection, the conjugate gradient method (CGM) which is closely related to the trust region method, has been identified as an effective solution technique for the class of optimization problems. This accounts for the intense research effort ongoing in this area.

As a result of its very low memory requirement the CGM has been efficiently utilized in solving large-scale optimization problems. We have identified a number of ways to improve the results obtained with the CGM for nonlinear optimization, namely, increasing the order of approximation, improving the accuracy of the conjugate gradient update parameter, refining the line search procedure to have a more appropriate step length, and forming hybrid methods.

Using the first approach, **Bamigbola** and Nwaeze (2006) proposed a high-order representation to replace the quadratic model of the conventional CGM. The high-order nonlinear model possessed desirable features that made it a veritable tool for analysis and application in the area of gradient-based optimization.

Based on the second approach, **Bamigbola**, Ali and Nwaeze (2010) developed a new CGM, known as Bamigbola-Ali-Nwaeze method, for unconstrained large-scale optimization. The result was presented at the 2010 International Congress of Mathematicians at Hyderabad, India. Some of the desirable properties of the new method include sufficient descent and global convergence properties. Numerical results showed that the new method is efficient in computation and superior to other similar methods.

To refine the line search procedure in order to enhance efficiency with the CGM, two tactics are in vogue. The more common is the line search technique, while the more recent utilizes a formula. Line search techniques usually require many evaluations of the function and gradient expressions, thus, reducing computational proficiency compared to the latter technique, where such evaluations are not needed. The first unified step-size formula for the CGM reported in literature was not suitable for large-scale optimization as it involved the evaluation of some matrix entity.

Later, another fixed formula which is matrix-free was obtained, but the step-size formula depended on some parameters whose choice of values are more or less arbitrary. Removing the arbitrariness in assigning values to the parameters was one of the objects of the third proposal. In enhancing the more recent step-size formula, explicit representations were made for the parameters by the application of Taylor's theorem to the multivariable objective function, and this effort yielded two new step-size formulae. The paper titled "A gradient-based step-size rule for conjugate gradient methods" published in pages 85-92 of volume 35 of Journal of the Nigerian Association of Mathematical Physics by the duo of Ajimoti and **Bamigbola** (2016a), detailed on the better of the two step-size formulae.

Through some extensive numerical experiments, Ajimoti and **Bamigbola** (2016) in the Inter-national Journal of Applied

Science and Mathematical Theory, demonstrated the superiorrity of the Ajimoti-Bamigbola (gradient) rule over the other existing step-size formulae and line search techniques in terms of the number of iterations and execution times. Later, **Bamigbola**, Ajimoti and Dennis (2020) established theoretically the global convergence of the two step-size formu-lae, and numerically validated them on a set of large-scale unconstrained optimization problems drawn from the CUTEst Suite, available at http://www.cuter.rl.ac.uk/Problems/mastsif.shtml.

In solving unconstrained optimization problems, both Broyden-Fletcher-Goldfarb-Shanno (a variant of the quasi-Newton methods), and the CGMs were known to be efficient. Hence, the optimal hybrid Broyden-Fletcher-Goldfarb-Shanno-Conjugate Gradient (OBFGS-CG) method was proposed to combine the strengths of both BFGS and CG methods. The optimal hybrid BFGS-CG method was based on an existing hybrid BFGS-CG method in which the optimal parameter was substituted. The OBFGS-CG utilized in solving unconstrained optimization problems, gained an improvement in the total number of iterations and CPU time. This work was reported in **Bamigbola**, Okundalaye and Ejieji (2018).

Another approach at enhancing the efficiency of the CGM is by the application of mathematically broad-based concepts, one of which is the notion of linear combination of vectors. Akinduko and **Bamigbola** (2021) adopted the development of a generalized form of the classical CGM by forming a rational expression of the linear combinations of the numerator and denominator terms in the update parameters of ten existing classical CGMs. Combinatorial analysis was carried out to establish that the new classical methods produced were about one hundred times the number of existing CGMs. A numerical test of the new methods vis-a-vis the ten existing classical CGMs confirmed that some of the new CGMs are of superior computational performance with respect to the number of iterations and CPU time. This result is indicative that none of the existing CGMs is computationally optimal in efficiency.

Hence, there is a need to explore the generalized CGM for the best possible classical CGM.

Optimal control problems

A great deal of theoretical work on parabolic control problems has been reported in the literature. The approach most of the scholars employed to attack the problems delved much on mathematical abstraction and thus, lacks applicability. **Bamigbola** (1997) considered a direct approach that used the Ritz-penalty method to transform the original control problem into a variational form. Discretization, utilizing the finite element routines, resulted in a computational scheme that improved on accuracy, and at the same time minimizes computing time.

In Ogundele and **Bamigbola** (2005), a penalty function method was considered for the solution of the one-dimensional optimal control problem subject to a dynamic constraint. The area of application is widened by its extension to resolve dynamic constrained optimal control problems. The result obtained converged identically to the classical Euler-Lagrange method.

Bamigbola and Ejieji (2006) proposed the CGM via the Lagrange penalty method for equality constrained optimization. Although the conventional CGM is elegant in presentation, smooth in analysis and efficient in computation, it suffers major setbacks of being of low order in accuracy and applicability. As a result, the formalism of the higher order CGM was followed resulting in enhanced accuracy.

In a related effort, Ejieji and **Bamigbola** (2006) successfully extended the CGM, by incorporating the Lagrange imbedding earlier reported, to cope with both discrete and continuous optimal control problems. The results obtained with the proposed method not only compared favourably with those of some existing methods, but has the advantage of solving non-quadratic problems.

Adewale and **Bamigbola** (2015) contributed a chapter on global control theory and its applications to a book titled

"Control Theory: Perspectives, Applications and Developments" and published by Nova Science. Methods of control improvement were discussed including computer methods or iterative techniques of control improvement, namely, the gradient and needle methods. The new idea of the theory of optimal control makes use of the approach based on sufficient conditions for global optimality of control processes and a mathematical technique for global estimates related to it. An interesting characteristic feature of the global estimates technique was the richness of the analytical medium created by this technique. Some areas of application were cited as the control of the pure inertia plant, sliding mass, sliding regimes, and electricity generation.

As a result of water pollution, there is a growing interest in filtration, purification, and decontamination processes in urban areas to forestall outbreak of diseases and other harms that may come with the anomaly. To evaluate the impact of the remediation, the first step was to build mathematical models to study the aspect of our interest. All the models encountered in the study evolved as coupled or uncoupled partial differential equations, constrained or unconstrained. Getting an assurance that the solution at least exists and is unique was the next phase before selecting a solution procedure and finally implementing the obtained solution. It should be mentioned that in a number of instances it suffices to just control, monitor or guide the process.

Bamigbola and Agusto (2003) established the existence and uniqueness of an optimal solution to the constrained instationary incompressible linearized Navier-Strokes equations. Agusto, **Bamigbola** and Layeni (2004) used the semigroup approach to prove the existence and uniqueness of solution to the filtration problem, and established the existence of the optimal control and the optimal state.

Agusto and **Bamigbola** (2005a) coupled the filtration model to the heat equation, and by means of variational formulation, the conditions for the existence and uniqueness of solution to the coupled system were determined.

The application of a distributed parameter control for a diffusive-convective population, referred to as an optimal control problem for a nonlinear Dissolve Oxygen/Biological Oxygen Demand system with logistic growth term, was studied in Agusto and **Bamigbola** (2005b) for the purposes of mathematical model formulation and analysis of its state system. The existence and uniqueness of solution for the nonlinear system as well as the existence and uniqueness of optimal solution to the state equations were proved.

Wastewater contain a great variety of pathogenic bacterias and viruses. Control of biological pollution is usually done in terms of existence of faecal coliphorms because its concentration in domestic wastewater is much greater than that of other microorganisms. After discharge, concentration of bacterias or viruses decrease very quickly due to unfavorable conditions like lack of nutrients, low temperature, sun rays, etc. In what followed, Agusto and **Bamigbola** (2007a) considered a very simple and useful model for bacterial transport in polluted water bodies, established the uniqueness and existence of solution to the contaminant transport problem, and then stated the optimality conditions for the optimal control problem.

The role mathematical models play in evaluating the environmental impact of pollution, and predicting the pollution level in the regions under consideration cannot be over emphasized. Thus, the various mathematical models involving water pollutant, and their significance were examined in Agusto and **Bamigbola** (2007b). A computational approach for the evaluation of the generalized transport equation gave the implicit central difference scheme in space, and a forward difference method in time, which on implementation with the Crank-Nicolson numerical scheme while varying the different parameter values, gave profiles for different factors in the transport equation.

Agusto and **Bamigbola** (2008) studied a domain occupied by shallow water of polluted wastewater with a mandate to choose some indicators of pollution levels so as to simulate the water quality in the domain. The conditions to be satisfied for a unique solution to exist for the nonlinear system as well as the conditions for existence and uniqueness of optimal solutions were determined. This important result guarantees the attainment of a required level of water quality, and was published in the *Applied Sciences*.

Abstract optimization

The general optimization problem can be studied abstractly, devoid of any computation, by merely prescribing the conditions to be satisfied by the objective function on the domain satisfying certain properties. The essence of optimization abstraction is for ease of characterization of the solution. The research works discussed in this section took into consideration the nature of the global or local, and the type of the domain of the problem, whether finite-dimensional or infinite dimensional. Both global optimization and infinite-dimensional problems are difficult to resolve.

The enormous difficulty inherent in global optimization problems is that all standard techniques in nonlinear programming can at most find local optima. Fortunately, certain classes of mathematical programs have the property that all local optima are global optima, a well-known example being convex problems, where the objective functional and the feasible domain is a convex set. Thus, Osinuga and **Bamigbola** (2004) utilized the concept of convexity and the projection principle to characterize the solution of multi-extremal global optimization problems on abstract spaces.

Bamigbola, Makanjuola and Adelodun (2006) utilized the R-multiplier to fashion abstract optimal solution to mathematical programming problems. The versatility of the abstract tool was demonstrated in identification of its mathematical features of symmetry, domain extension and continuity properties. The study demonstrated the intricate link between the abstract field of semigroup and optimization in applied mathematics that pervades almost the entire sphere of human endeavor.

The Weber problem, often referred to as the minimum norm problem, is of utmost practical application in different fields of engineering, science and technology. The problem has two very important properties. First, *the objective function* is convex, which ensures that any local optimum is also a global optimum. Second, the optimal location for the new facility must lie within the convex hull of the existing facility locations. Based on these characteristics, Osinuga and **Bamigbola** (2007) propounded that the objective function is a Lipschitzian function, which is necessarily a convex function, and established the existence of global minimum norm solution using coercivity conditions.

In a similar effort having to do with unconstrained optimization on reflexive Banach spaces, Osinuga and **Bamigbola** (2008) obtained and characterized a set of optimality conditions using compactness and continuity properties to identify the local solution.

Noting that the modern theory of optimization in normed linear space is largely about the relationships between a space or domain and its corresponding dual, **Bamigbola** and Osinuga (2013) considered the general optimization problem over the Hilbert space. The criterion for existence and uniqueness of solution were specified by formulating the minimum problem in the dual space. The optimal solution was characterized by the abstract space $C^p[a, b]$, $1 \leq p \leq \infty$.

Importance of infinite-dimensional optimization cannot be undervalued especially for practical applications. Typical examples include minimal surface problems, boundary control problems, image processing and semiconductor design. Osinuga and **Bamigbola** developed existence and uniqueness theorems for the infinite-dimensional optimization problem and applied the result to the space of continuously differentiable functions.

Industrial Optimization

Mr. Vice-Chancellor Sir, Ladies and Gentlemen, my research activities in this section will be presented under three broad applications in the critical industrial segments, namely the power, financial and health sectors. In dealing with real-life or physical problems, mathematical modelling is always of great advantage because of its power to predict system behaviour and provide a clear insight of the important inputs and outputs.

Electric Power System

Energy is a basic necessity for the development of any nation. Although, there are different forms of energy, the most important of them is electrical energy, Rajput (2003). Electricity is the basic necessity for economic, industrial and social development of any nation, and is a measure of the advancement of a country.

It is instructive to note that the electricity industry is operated as the electric power system, which can be divided into the generation, transmission and distribution components as illustrated in Figures 8 - 10. The goal of an electric power system is to produce electric energy in sufficient quantities at the most suitable generating locality, transmit it in bulk quantities to the load centres, and then distribute it to the customers in proper form and quality and at the lowest possible ecological and economic price, Shahidehpour and Abbasy (1988).

Figure 8: Electric power generating stations
Scource: Google.com

Figure 9: Electric power transmission subsystems
Source: Google.com

Figure 10: Electric power step-down station
Source: Google.com

Our moderate contributions to the generation and transmission subsystems are now presented.

Electric power generation

The factors influencing power generation at minimum cost are operating efficiencies of generators, fuel cost and transmission losses. The most efficient generator in the system may not guarantee minimum cost as it may be located in an area where fuel cost is high. Also, if the plant is located far from where the load is, transmission losses may be considerably higher and thus, the plant may be uneconomical. Hence, the problem is to determine the generator output for different plants such that the total operating cost is minimal.

Bamigbola and Aderinto (2009) in a bid to model the electric power generation module, recast the above objective statement as:

Find the controls u, v that minimizes the cost function

$$J(u,v) = \int_{t_0}^{t_f} [\tau^T C(t) + \eta\, u^T u + \mu v^T v] dt, \qquad (4)$$

subject to

$$\frac{dG(t)}{dt} = \alpha + A_1 C(t) G(t) + A_2 u(t) G(t) - \beta G(t) \qquad (5)$$

$$\frac{dC(t)}{dt} = (s+y) + B_1 C(t) G(t) - B_2 v(t) C(t) + \gamma C(t) + \delta G(t) \qquad (6)$$

and

$$G(t_0) = G_0, \ C(t_0) = C_0,$$

where $\tau = (\tau_1, \tau_2, \dots, \tau_m)$ is the unit expenditure on the generators, η and μ are parameters to balance the size of the controls, α is the initial generator output, s+y is the sum of labour and maintenance costs, $\beta = (\beta_1, \beta_2, \dots, \beta_m)^T$, $\gamma = (\gamma_1, \gamma_2, \dots, \gamma_m)^T$, $\delta = (\delta_1, \delta_2, \dots, \delta_m)^T$,

$$A_1 = \begin{pmatrix} q_1 & 0 & . & . & 0 \\ 0 & q_2 & 0 & . & 0 \\ 0 & 0 & q_3 & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & q_m \end{pmatrix}, \quad A_2 = \begin{pmatrix} k_1 & 0 & . & . & 0 \\ 0 & k_2 & 0 & . & 0 \\ 0 & 0 & k_3 & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & k_m \end{pmatrix}, B_1 = \begin{pmatrix} r_1 & 0 & . & . & 0 \\ 0 & r_2 & 0 & . & 0 \\ 0 & 0 & r_3 & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & r_m \end{pmatrix},$$

and

$$B_2 = \begin{pmatrix} x_1 & 0 & . & . & 0 \\ 0 & x_2 & 0 & . & 0 \\ 0 & 0 & x_3 & 0 & 0 \\ . & . & . & . & . \\ 0 & 0 & 0 & . & x_m \end{pmatrix}.$$

In a related work, see Aderinto and **Bamigbola** (2012), a qualitative treatment of the resulting mathematical model of the electric power generating system was undertaken. It was shown that there exists a unique optimal pair (u*, v*) for the optimal control problem provided t_f is small, which tallies well with the practice in the electricity industry. Analytical solution can be obtained for the optimal control problem via Riccati transformation.

Aderinto and **Bamigbola** (2013a) gave the optimal control model of the electric power generating system a quantitative treatment to complement the earlier results obtained. Precisely, the solution was obtained via the fourth-order Runge-Kutta scheme using real-life data obtained from the National Grid Centre at Osogbo, and it was observed that for efficiency and effective functioning of the generating machines in each station, monitoring of the control is very essential.

An alternative model was formulated for the electric power system in Aderinto and **Bamigbola** (2013b) with the goal to develop an optimum dispatch/generating strategy by presenting economically the best load flow configuration in supplying load demand among the generators.

Pollution is one of the challenges associated with electric power generation. Indeed, electric power generation is responsible for a large proportion of air pollution which is caused by burning fossil fuels, but the introduction of technologies can improve the efficiency of the power plant. Contributing to the measures to remediate the attendant contamination, Aderinto and **Bamigbola** (2017) proposed mathematical expressions to represent the dynamics of the air pollution via an optimal control theory approach. The model is characterized with the aid of Pontyagins maximum/minimum principles. The optimality system established the minimization of the cost of applying technology for efficiency improvement and the fine for atmospheric emission, while maximizing the electric power generation output.

A direct relationship can be shown to exist between (electric) power availability and per capital of a nation. The economy of a nation is easily identified with the amount of electricity it supplies. The advanced economy of the world generates a lot electric power and hence they witness rapid economical and industrial development. Third-world countries often witness epileptic or unstable supply of electric power. It is common knowledge that to have constant and stable electricity supply, a country needs to invest in its power generation.

The art of electric power generation as complex as it may seem can be aided with the use of mathematical modelling. The real-life data from power generating stations have been subjected to mathematical analysis for the purpose of improving the quality and quantity of electric power generated, and minimize the cost of production under secured conditions.

Other results on the aspect of electric power generation are Aderinto and **Bamigbola** (2014) and on controllability property of optimal control model of electric power generating system, Aderinto, and Bamigbola (2015).

To a layman, the results can be interpreted by saying that the electric power system can be expressed by using mathematical equations which relate two or more parameters that can be used to measure the state of the system. The parameters enable us to know the condition and capacity of the generators, how to use them, and how long to use them so as to maximize the outputs of the generators and minimize the cost of production.

From the discussion so far, the electric power sector has been portrayed as an area that needs to be studied as a multidisciplinary subject to harness contributions from relevant disciplines.

In a bid to simplify mathematical model of a system, and consequently enhance its performance, the strategy of reduction in the number of its factors is often applied, based on the profile of relative importance of the factors to the system. Parameter sensitivity analysis is an approach given to a model so as to define significance of the factors related to the model where the whole parameter space is fully described. In most practical problems, we are interested not only in the optimal solution of the control problem but also in how the solution changes when the parameters of the problem change. The change in the parameters may be discrete or continuous. The study of the effect of discrete parameter changes on the optimal solution is called the sensitivity analysis, while that of the continuous changes is termed the parametric programming, Rao (1979).

Latunde, **Bamigbola** and Aderinto (2016) carried out a sensitivity analysis of the parameters in the electric power generation system discussed above with a view to determine the relative importance and effect of each parameter on the model's results. The refinement of the model of the electric power generation system by means of maximizing the accuracy of the solution while at the same time minimizing the computational efforts and time required, provides enough justification for the reduction in the number of parameters.

Electric power transmission

The transmission system is to deliver bulk power from power stations to load centres and large industrial consumers, while the distribution system is to deliver power from substations to various consumers.

From the physics of electric power transmission, when a conductor is subjected to electric power (or voltage), electric current flows in the medium. Resistance to the flow produces heat (thermal energy) which is dissipated to the surroundings. This power loss is referred to as ohmic loss. Furthermore, if the applied voltage exceeds a critical level, another type of power loss, called the corona effect occurs. The power losses accumulate as the induced current flows and the corona effect propagate along the transmission lines. The power losses could take off a sizeable portion of the transmitted power since transmission lines usually span a long distance, sometimes several hundred kilometers. The overall effect of power losses on the system is a reduction in the quantity of power available to the consumers. Thus, adequate measures must be put in place to reduce power losses to the barest minimum.

Bamigbola, Ali and Awodele (2014) developed a mathematical model for determining power losses over typical transmission lines, as the resultant effect of ohmic and corona power losses, taking into cognizance the flow of current and voltage along the lines. The mathematical model for electric power transmission was obtained as:

$$\frac{\partial^2 I(x,t)}{\partial t^2} + (\lambda + \beta)\frac{\partial I(x,t)}{\partial t} + \lambda.\beta I(x,t) = \phi\frac{\partial^2 I(x,t)}{\partial x^2}, 0 < x < l, t > 0$$

together with

$$\frac{\partial^2 V(x,t)}{\partial t^2} + (\lambda + \beta)\frac{\partial V(x,t)}{\partial t} + \lambda.\beta V(x,t) = \phi\frac{\partial^2 V(x,t)}{\partial x^2}, 0 < x < l, t > 0$$

subject to boundary conditions

$$V(0, t) = V_0, \quad V(l, t) = 0, \qquad l \rightarrow \infty, t > 0,$$

where I denotes current, V, voltage, with λ and β as known constants.

By the application of Kirchhoff's current and voltage laws to an electrical circuit, treating a transmission line as a collection of infinitesimal lumped closed circuits, the above models of current and voltage flow on the electric transmission line were obtained. A discriminant analysis classified the models as second order linear partial differential equations (with constant coefficients) of the hyperbolic type, and may be called one-dimensional hyperbolic expressions embedding the wave equation.

Various solution methods were applied to solve the models. Oke and **Bamigbola** (2011) obtained analytical results for the transmission line equation, where leakage to the ground was very negligible, by making use of the Laplace transformation.

Utilizing the knowledge that in transmitting electric power, the elapsed time is very small, the transmission line equations reduce to:

$$\phi \frac{d^2 I(x)}{dx^2} = \lambda.\beta I(x),\ 0 < x < 1 \quad \text{and} \quad \phi \frac{d^2 V(x)}{dx^2} = \lambda.\beta V(x),\ 0 < x < 1$$

subject to boundary conditions

$$V(0) = V_0,\ \ V(l) = 0, \quad l \to \infty.$$

Analytical solution and actual data are shown in Table 2 and depicted graphically in Figure 11.

Table 2:
Available current and voltage along a 330 kV single circuit of a typical high voltage transmission network.

Length of line (km)	Current (A)	Voltage (kV)
10	19.14	329.5
20	19.09	329.1
50	19.00	327.6
100	18.87	325.3
200	18.60	320.7
300	18.33	316.1

Source: Bamigbola, Ali & Awodele (2014)

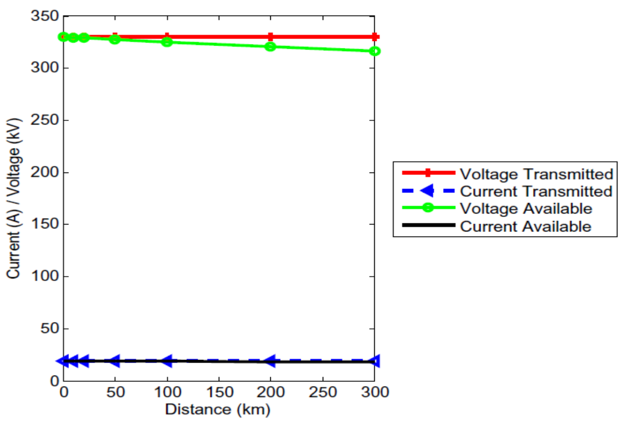


Figure 11: Transmitted and available current and voltage of the transmission line.
Source: Bamigbola, Ali and Awodele (2014)

Capitalizing on the boundary conditions by using an operator decomposition method, see **Bamigbola**, Oke and Aderinto (2021), gave more realistic results. The operator factorization technique was the basis of the solution method. Incorporating the initial and boundary conditions yielded the complete solution. The model was simulated using data obtained from a typical transmission station to deduce optimal values of key transmission parameters.

Formulating the power losses as a combination of the ohmic and corona losses, the problem of finding the optimum power losses on transmission lines was obtained as

$$Minimize\ T_L = I^2 \frac{\rho L}{A} + 242 \frac{(f+25)}{\delta} \sqrt[4]{\frac{A}{\pi d^2}} (V - V_C)^2 . (10)^{-5}$$

(I, V, d)

kW/km/phase,

with ρ as conductor's resistivity, L, length, f, frequency of transmission, A, cross-sectional area, δ, air density factor, d, distance between transmission lines and V_c, the disruptive critical voltage. Application of the classical optimization technique yielded the minimum point as (I^*, V^*, d^*) where $I^* = 0$, $V^* = V_c$ and $d^* = r.e^{\frac{U_p}{18r}}$ with U_p being phase voltage and r, radius of the conductor. Hence, for minimum power losses, the optimal strategy is to operate the transmission system at the disruptive critical voltage and letting the distance between the conductors to be larger than their diameter. The results are in line with current practices in the industry, except that of d^*.

Since power loss minimization is a very important area of concern in electricity transmission and even in distribution, there is a need to be able to predict the losses ahead of time. Thus, **Bamigbola**, Ali and Oke (2014) proposed mathematical models for predicting available current and voltage as well as the power losses along a typical transmission line so as to be able to reckon the net electric power available to be used to meet customers' demands. In the process, the evolution of current and voltage on the transmission line was studied and mathematical models to predict both current and voltage were constructed. In the end, the desired model for predicting power losses along transmission lines were formulated by reframing the power loss function as a mathematical physics problem. This strategy led to the exclusion of all the transmission parameters from the model. Numerical results obtained are presented in Tables 3 and 4.

Table 3: Predicted power losses for a 330 kV single circuit of the typical transmission network.

Length of line in km	Power losses (in MW) for a load of 100 MW $\alpha = 0.00005$	Power losses (in MW) for a load of 200 MW $\alpha = 0.00009$	Power losses (in MW) for a load of 300 MW $\alpha = 0.00014$
10	0.0500	0.1800	0.4300
20	0.1000	0.3598	0.8594
50	0.2500	0.8983	2.1438
10(	0.5000	1.7927	4.2724
200	0.9953	3.5694	8.4839
300	1.4892	5.3301	12.6354

Source: Bamigbola, Ali and Oke (2014)

Table 4: Simulated results of power losses on 330 kV single circuit of the Nigerian transmission network.

Length of line in km	Power losses (in MW) for a load of 100 MW	Power losses (in MW) for a load of 200 MW	Power losses (in MW) for a load of 300 MW
10	0.05	0.18	0.43
20	0.09	0.37	0.87
40	0.18	0.73	1.75
60	0.26	1.10	2.84
100	0.41	1.85	4.66
200	0.76	3.77	10.86
300	1.10	5.85	24.40

Source: Bamigbola, Ali and Oke (2014)

Application of the classical optimization technique aided the formulation of an optimal strategy for minimization of power losses on transmission lines. With the aid of the new models, it is possible to determine current and voltage along the transmission lines. In addition, we note that the analytical method does not involve any design or construction and so is less expensive than other models reported in the literature. Hence, the goal of the very well-known engineering problem – reducing the power losses on transmission lines to the barest minimum – has been achieved.

PV solar power

Solar energy technologies offer a clean, renewable and domestic energy source, and are essential components of a sustainable energy future. In the design and evaluation of solar energy and power, information on solar radiation and its components at a given location are needed. In this regard, solar radiation models are of big importance. As a result, **Bamigbola** and Atolagbe (2021) developed models of global solar radiation using information of some easily accessible factors, available from repository of solar radiation data, and utilize the models in measuring PV solar power outputs. By means of interpolation, global solar radiation models were developed using data of monthly average insolati on from the National Aeronautics and Space Administration (NASA) for the period of 1996 - 2017. The identified factors were considered as input variables in formulating global solar radiation models which were presented in closed form for the five subregions of Africa. The novelty in the study was that accurate, reliable and computationally less burdensome empirical models, with a few input factors, were developed for use as predictive tools in estimating global solar radiation derivable for any location. The study also affirmed the latitude as the dominant locational factor. In addition, the domain of application in the study is the largest so far. Finally, the new models featured optimal performance in respect of estimating global solar radiation for any practical application.

Later, Atolagbe and **Bamigbola** (2021) formulated models for PV solar system outputs in terms of current and voltage based on such factors as ambient temperature, humidity, and wind speed. The PV solar system models were obtained as multivariable functions of the input factors through symbolic regression via genetic algorithm using Eureqa software. The models were validated using 18 years of average of meteorological data for two locations in the selected base for the study. The analysis performed on the results from the two approaches showed that the models can accurately predict power output from PV solar power systems since models prediction errors were in acceptable range. By this effort, a major challenge of providing reliable electricity data for standalone and grid connected PV systems has been overcome. As such, the results of the study can be utilized in the efficient management of solar power generation.

Epidemiology research

My interest in epidemiological studies, as an analyst, was because the area involved a lot of logical and computational skills that are best handled by the mathematically inclined.

Infectious diseases are the most dangerous and killer entities worldwide, and they pose the most significant health challenges facing the global community.

Vaccination is always the approach recommended by health workers and epidemiologists to employ in controlling communicable ailments. But, finding the best way to vaccinate people against infectious disease is an important issue for health workers. In seeking an informed response to that important matter, Bolarin and **Bamigbola** (2014) formulated a six-compartmental model with pulse vaccination and saturated incidence, to study the dynamics of infectious disease in a population. The conditions in compact form, that guarantee the existence of the disease-free periodic solution was established. From the study, it was discovered that short pulse vaccination, long latent period or long immune period will guarantee eradication of the disease in the population. Lastly, applying the optimal control theory to the newly developed model, the cost benefit of maximizing the recovered individuals and minimizing the infected and susceptible individuals were then determined.

A joint effort between **Bamigbola** and Fowosire (2014) on prediction and control of Avian Influenza, another highly contagious disease, can be aptly captured as follows: Avian, human and bird, influenza co-existence model was designed and used as basis for the study of the transmission dynamics of the disease across the bird and human populations. The models were used to obtain the various reproduction numbers of the influenza strains, and the invasion reproduction numbers of one strain over another were formulated. H5N1 human cases and its cumulative correspondent data were drawn from the World Health Organization (WHO) database for a period of 12 years starting from 2004. Through curve fitting, predictions for the model were made. At the end of the study, it was observed that the predictions closely tallied with the parameters of the model. The curve fitted to the data was used to make future prediction of a possible outbreak. Lastly, the current control measures and their level of efficiency based on the elasticity of reproduction numbers that avian influenza best control mechanism is through culling for birds and quarantine for humans.

Recent efforts, some of which are ongoing, have to do with developing data-driven models to study the spread and control of communicable diseases. Specifically, **Bamigbola** and Akindoyin (2021) proposed a diffusion-reaction model to characterize the dynamics of meningitis. A numerical approach was considered for the solution of the problem. With the aid of data generated from an instance of meningitis outbreak, the rate of spread of the disease and the quantity of treatment to apply to control the disease were determined.

Financial system

The financial sector is at the heart of any nation as that division takes care of the monetary, fiscal and economic policies. Included in this presentation is my research engagements in a special segment of the financial sector.

Latunde and **Bamigbola** (2016a) examined some issues having to do with the management of net risky capital assets. A new optimal control model of asset management for a business organization was proposed based on the uncertainty theory in which the capital assets are managed. In addition, the problem was solved analytically using principle of optimality, and the optimal controls obtained.

In some further studies on asset management by Latunde and **Bamigbola** (2016b), uncertain differential equation was used to model a special case of asset management where the investor was interested in determining the expected net asset such that the net worth would be maximized. Conditions for the existence and uniqueness of solution to the asset management problem were derived with respect to Lipschitz criterion and linear growth measure.

In order to demonstrate one of the essential features of the optimal control model of asset management, Latunde and **Bamigbola** (2016c) developed a number of stability theorems, with proofs, to give assurance that the model adequately represents the settings of the net risky capital asset.

The research work reported in Latunde and **Bamigbola** (2018) was applied to a problem of capital asset management which dealt with the debt crisis of our nation's economy. The exercise was carried out using real-life data in order to provide control policies in resolving the debt crisis and also to show the satisfaction of optimality criteria. Numerical solutions were obtained with the aid of some numerical scheme for the discretization of both the objective functional and the uncertain differential equations. Finally, the optimal control decision policy was obtained and discussed.

Research in Computing Mathematics

The efforts expended in the processes of course registration of students and computation of their examination results in tertiary institutions are awesome. Quite worrisome is the fact that these processes are carried out every academic session, putting the operators in a continuous and everdemanding cycle. The computation of examination results and course registration of students are obviously object-centred activities, the student being the dominant object in this case. Thus, the need to evolve not just a computerized system, but an object-oriented software design and implementation that will effectively and efficiently capture all important objects associated with the course registration and examination results processing within the tertiary institution. This genuine and noble desire, spearheaded by me in the year 2001, led to the design and development of a software package acronymed SPERU (Undergraduate Registration and Examination Processing System) by **Bamigbola**, Olugbara and Daramola (2004). SPERU was demonstrated as a well-documented software system in-built with modules for capturing student biometrics, and that testified to the beauty, power and supremacy of the object-oriented paradigm. The results of applying software predictor metrics characterized the package as reliable, maintainable, extendible, portable, efficient and secured. It is important for me, at this junction, to gratefully acknowledge the gracious adaptation of SPERU for use in our 'Better by Far' University by the authorities of the university. Later versions of the software are still in use in our university.

The advent of object-oriented paradigm has greatly advanced the efficiency and accuracy of software modelling of real-world problems. **Bamigbola** and Daramola (2004) reviewed the basic tenets of the object-oriented model and presented a mathematical formalism for its features. This proposition of modelling objects as algebra attests to the maxim that "Mathematics is the root of all modern discovery in science", and thus linking concepts in software engineering and abstract algebra to advance the frontiers of knowledge in technology. An application of the concept was successfully demonstrated, and areas where the postulations of object-orientation will gain increasing relevance, in the future, were highlighted.

Historically, software development has always been a great challenging enterprise as a result of the need to optimize cost, time and accurate realization of software requirements to satisfy the growing sophistication of user demands. In proposing the dynamic system development method for resolving the highlighted challenges, **Bamigbola** and Daramola (2005a) identified the essential attribute that makes the approach a veritable tool for boosting the quality assurance metrics of software products. Also, a case study of the dynamic system development approach, as employed by a Nigerian university in developing a software application for student course registration and examination results computation, was reported. The paper was concluded with some of the benefits of the proposed method, and made recommendations for all stakeholders in the software development industry.

Bamigbola and Daramola (2005b) observed that outsourcing of software development processes by bigger companies, mostly in the developed countries to the smaller companies of less developed economies, has become the order of the day. The rise in globalization, and the advent of the Internet, where virtually every kind of information is made available are major catalysts. Outsourcing is the process of placing the development of certain software tasks in a software development project in another development organization. The general trend in these arrangements, in most cases, was for an organization to place the outsourcing subcontract in places where it could get high competence at a reduced cost, which could be in terms of cost of labour or other development costs. The Asian countries and most notably India have benefited tremendously from these arrangements. We believe that Africa and most notably Nigeria can also draw from this software boom. Developing countries of the world like Nigeria, and Nigerian software developers also have the potentials to partake of this prospect if concerted efforts are made to be correctly positioned as outlined in the paper.

Limited infrastructures and resources for training and learning at the tertiary level in many third world countries has resulted in instances of overcrowding of lecture rooms and reduction of staff to student ratio which has further depleted the existing academic standard. A potential antidote is the adoption of Web-based learning, which has proved relevant to both distant and on-site learning. Thus, Daramola and **Bamigbola** (2005a) introduced an integrated framework model for web-based learning. The framework architecture was made up of an online classroom, online result checking, online course assessment and online course registration components. The framework model was presented as an instance of novel integration of component-based development, multimedia streaming technology and web services based on a reliable software architecture. As an on-going project then, it was expected to be a successful demonstration of the software product-line engineering initiative and a worthwhile case study for other practitioners in the field of software engineering practice. The emerging products were also expected to be reliable web-based learning systems that would lift the standard of tertiary education. It is important for me to point out that the prophecy has come to pass with the advent of virtual learning, ceremony, etc in our country.

Although component-based software development and component technology offer exciting and interesting potentialities for the simplification of complex software models in order to gain increased functionality, reduction in time and cost of development, the desire to enhance the user's understanding and perception of components' behaviours was still a pressing challenge. Daramola and **Bamigbola** (2005b) opined that since users do not have access to the source codes of components developed in most cases, it is necessary for developers to give a detailed specification of component functionalities, behaviour and performance in a way that will enhance their predictability even before they are deployed. As such, the semantics of the CSP-like WRIGHT were adopted in the specification of a visual component tool named "DB-Data Enkrypta" that can be used to compress and encrypt texts stored in a relational database. The developed tool was recommended for the use of component developers as it has the tendency to greatly improve the understanding and perception of component behaviour, boost component interoperability in order to minimize misuse and mismatches in the composition of component-based system.

The ever growing requirements of users for more advanced functionalities in systems have made the software development process more complex. This has also created the need for higher-level abstraction of software systems in order to accurate and sufficiently model real-world systems. Daramola and **Bamigbola** (2006) highlighted the attraction and challenges of software componetization, and gave a report of the successful application of the concept in the design and development of an automated three-tier component-based software, christened 'Auto-budget', for recurrent budgeting and planning in a tertiary institution.

A measure often used to evaluate the performance of algorithm is execution time, which is a function of some other computer metrics. Thus, attempt to compare algorithms solely on the basis of computer time is faulty. Okeyinka and **Bamigbola** (1999) therefore suggested evaluating algorithms based on their own merits, through the measure of the number of elementary operations required by the different algorithms. A software tool, called 'Scanner' that counts the number of elementary operations in a program to compute the algorithm's complexity was designed, developed and implemented. Though developed in PASCAL, the scanner, with little modifications, can be turned into a universal machine.

Halstead complexity metrics are a means of determining a quantitative measure of complexity directly from the operators and operands in a procedure. However, in doing so the traditional Halstead regarded the complexity of the sequential procedure, and that of the recursion to be equal. In other words, the depth of recursion is not taken into consideration while computing the complexity of the recursion. Okeyinka and **Bamigbola** (2012a) studied the depth of recursion by translating it as iteration using stack data structure. The results showed that contrary to the traditional belief, a recursive function is more complex than its equivalent non-recursive counterpart.

Complexity of algorithms can be studied analytically using the concept of Big O notation. One of the flaws of such a study is that the complexities obtained for algorithms are in most cases the same; whereas in reality such algorithms might vary in terms of efficiency. The reason for the disparity is due to the definition of the Big O itself which treats all arithmetical operations as behaving the same way, whereas this is not so. For example, with the Big O concept, addition and multiplication are expected to be carried out in exactly the same way, but factually, multiplication is expressible as repeated addition. Thus, for pragmatic purposes, there is a need for estimating actual complexities of each algorithm to be sure which one is the best, given more than one algorithm for solving the same problem. Okeyinka and **Bamigbola** (2012b) conducted complexity experiments on a number of algorithms for solving the same set of problems. The findings were that numerical solution of complexity of algorithm gives more detailed, more specific and hence more pragmatic results than those obtained analytically and that recursive algorithms are more complex and hence, less efficient than non-recursive algorithms.

Strings are the most general medium for the representation of information, and which may be collected in a repository for purposes of information exchange. As such, strings and databases are germane to concepts in bioinformatics. An important issue critical to the analysis of complex information is the pattern matching problem. Oladele, **Bamigbola** and Gbadeyan (2008) highlighted the practical significance of the pattern matching problem as common computer applications in word processing, textual information retrieval programs, internet browsing, and the like. Furthermore, the suffix tree was presented as an efficient data structure that provides efficient access to all substrings of a string, and is able to rapidly align sequences containing millions of nucleotides and give significant biological information. Hence, the suffix tree data structure was used to solve cognate pattern matching problems.

The alignment of sequences is a mutual arrangement of two or more sequences which exhibit their similarities and where they differ. Sequence alignment is usually used to study the evolution of the sequences from a common ancestor, especially biological sequences such as protein sequences or deoxyribonucleic acid (DNA) sequences. Sequence alignment can also be used to study the evolution of languages and the similarity between texts. The typical assumption in the use of alignment is the mechanism of molecular evolution in which DNA carries over genetic material from generation to generation, by virtue of its semi-conservation duplication mechanism. Oladele, **Bamigbola**, and Bewaji (2009) exposited on sequence alignment algorithms for finding similarities between sequences, homologues (relatives) on a gene or gene-product in genomic databases. The information is useful for answering a variety of biologically related enquiries.

Research in Applicable Mathematics

Mr. Vice-Chancellor Sir, reported below are my efforts in diverse real-life applications, otherwise referred to as applicable mathematics.

Consider a salesman who is to visit a specified number of towns, for which he is required to call at each town only once before returning home. How should he schedule his visit so as to travel as little as possible? This is the travelling salesman's problem. It is a famous combinatorial problem of very wide practical application in a variety of situations. The travelling salesman's problem is still being critically studied because no exact algorithm has been found for its solution. So all along recourse has been to the use of heuristics. Okeyinka and **Bamigbola** (1999) reported the discovery of a more realistic travelling salesman's problem, referred to as the constrained travelling salesman's problem, and a new heuristic to solve it.

The new heuristic was observed to perform better, in some cases, than its classical proponent.

The work report in **Bamigbola** (1997) was motivated by the need to resolve two practical challenges in the judicious utilization of human, natural and material resources. The first was at my place of work – the Department of Mathematics, University of Ilorin, and the other at the Regional Headquarters of my church - Apostolic Faith Mission, Anthony, Lagos, Nigeria. In order not to detract from the issues involved, the challenges are stated as follows:

Scenario 1:

The final year students in a non-specialized programme are to be allotted for project supervision. How can the students be grouped so that the sets of students under the various supervisors are of about the same ability?

Scenario 2:

A religious organization that holds an annual convention at its regional headquarters has a combined choir and orchestra whose membership cut across the different geographical locations in the region. Of recent, the musical segment of the church witnessed a membership explosion resulting in inadequacy in the number of seats on the performing stage. To resolve this challenge, it was decided that the musicians be grouped so that each member can officiate not more than a specified number of times a week. Propose a schedule for the performing groups that will reflect all the voice parts in the choir, the different classes of musical instruments, as well as have a good spread in terms of the geographical location, gender and age experience of the members.

In the study, it was observed that the two scenarios were related and are typical of myriads of problems confronting the society at large. The first problem was identified with the static programming, since it involves only one variable – academic ability of the students. On the other hand, there were five variables – voice part, instrument type, geographical location, gender and age experience – in the second scenario, hence, it was classified as a dynamic programming problem.

To tackle the problems, a homogenous distributor constructed in the finite-dimensional space, characterized with salient features and shown to yield the theoretical optimal solution. Then, the skill of mathematical programming was utilized to develop a scheme which was implemented in PASCAL, and numerical values that validated the analytical solution of resolving the identified real-life challenges, were obtained.

In furthering research efforts to enhance the operation and outputs of the weaving industry, **Bamigbola**, Adeyemo and Rauf (2004) developed some mathematical models which served as basis for solving the problem of seeking appropriate values for certain variable parameters in the weaving process. The findings of the study indicate that if any parameter is accurately calculated and substituted into the models, weaving outputs such as the tension for vertical loom, amount of time and the number of yarn to use to produce the exact fabric requested by customers can be determined. Another output is the total cost of producing a quality fabric, thereby eliminating wastages as well as enhancing the economical and occupational benefits of the trade and industry.

My Contributions to Teaching, Administrative and Community Service

Mr. Vice Chancellor Sir, distinguished Ladies and Gentlemen, apart from engaging in research activity, a university academic is also expected to be involved in teaching as well as administrative and community service. My active participation in these other areas are presented below.

Contributions to Teaching and Academic Enhancements

For more than 30 years, I have been actively and continuously teaching in the university system. I have taught in not less than four Nigerian universities either as full-time, visiting, adjunct or part-time staff. Last year, before the COVID-

19 induced lockdown, I was a participant under the International Mathematical Union/Community of Developing Countries Volunteer Lecturing Programme in a university in Kampala, Uganda in pursuit of the mandate to disseminate knowledge in the area of mathematical sciences. I have taught a number of courses in the areas of pure and applied mathematics to undergraduate and postgraduate students.

There is a dual relationship between teaching and research. A good teaching career is a plus for research endeavours, while activeness in research, as a matter of experience, stimulates the spirit of inquiry which also imparts positively on teaching.

I seize this opportunity, to inform this assembly that most of those research results presented earlier have appeared in scholarly peer-reviewed and reputable publication outlets. On the Google scholar web, eighty-five of my research publications has been credited with 197 citations, an h-index of 8 and i-index of 5.

My search for knowledge in my areas of expertise has afforded me some opportunities including:

i. Supervision of 30 Masters dissertations and 16 PhD theses successfully to completion, defense and award.
ii. Regular attendance at local, national and international conferences for academic interactions.
iii. Honouring invitation as facilitator and guest speaker at workshops and conferences.
iv. Mentoring early career faculty members in Nigerian universities.
v. Visit of Centres of Excellence in Mathematical Sciences within and outside Nigeria..
vi. Attraction of foreign academics and fund for academic activities:
    - At least two of my research collaborators, Professors Montaz M. Ali of the School Computer Science and Applied Mathematics, University of

Witwatersrand, Johannesburg, South Africa, and Prof. Christopher P. Thron of the Department of Science and Mathematics, Texas A&M University-Central Texas, Killeen, USA, have visited the University of Ilorin to participate in some academic functions.

- Applicant and contact person for a £10,000 grant to mentor African research in mathematics at the University of Ilorin through the African Mathematics Millennium Initiatives (AMMSI) and London Mathematical Society. The grant was utilized between 2009 and 2011.
- Grant of €883.71 by the International Mathematical Union-Commission for Developing Countries for Volunteer Lecture Program at Kampala International University, Uganda, between 10th February and 12th March 2020.
- Scholarship award to attend International Congress of Industrial and Applied Mathematicians at Valencia, Spain in 2019.
- European Mathematical Society-Simons for Africa Grant for Collaborative Research 2018. A travel grant of $1,300 for research visit to the University of Kansas, Lawrence, USA from 27th September - 5th December, 2018.
- Research award by the (AMMSI). A competitive research grant to the tune of $1,850 supplemented with ZAR 3,000 by the University of Witwatersrand, Johannesburg for accommodation expenses during the research visit from 2nd October - 30th November, 2009.

Contributions to Administrative and Community Service

By the grace of God, I have served in various capacities in the university system. Some of the positions include membership of the University of Ilorin Senate, headship of department, chairmanship and membership of many committees at university, faculty and departmental levels, co-ordinatorship of academic programmes and departmental examinations as well as level and student associations' advising. Itemized below are some of my engagements in community service at cognate levels.

Academic services:

External examination of programmes in some Nigerian tertiary institutions, external evaluation of M. Sc. dissertations and Ph. D. theses, external assessment for promotion and appointment to the professorial cadre, participation in the National Universities Commission's accreditation and resource verification exercises, reviewing manuscripts for research journals, among others.

Professional services:

My professional affiliation include: Fellowship of the Mathematical Association of Nigeria (fellow and life member), Association of Mathematical Sciences and Optimization (Scientific Committee member), and member of the following: Nigerian Mathematical Society, Nigeria Computer Society, African Mathematical Union, International Mathematical Union, American Mathematical Society (Affiliate member), and Society for Industrial and Applied Mathematics.

Specific activities include providing administrative support to the Mathematics Improvement Programme in Secondary Schools of Kwara State Chapter of the Mathematical Association of Nigeria, serving as Chairman of the Association, consultant to Hope Rising Publishing Company, Ibadan and Sky Hope Engineering Limited, Zaria, and facilitator, Optimization Manpower Workshop at the Nigerian Paper Mill, Jebba.

Other communal services:

Participation in some activities of Omu-Aran Government Secondary School Old Boys' Association (OGSSOBA) including presentation of their inaugural address, membership of Agba Dam Estate Landlord Association, service as chairman, Board of Governors of Apostolic Faith Secondary School, Egosi-Ile, proprietorship of Topmark Academy (an educational institution), and serving as on the Board of Trustees of Crawford University, Igbesa, Ogun State.

Conclusion

Mr. Vice-Chancellor Sir, Ladies and Gentlemen, my presentation has demonstrated some truths about the attributes of Mathematics. Though Mathematics has been subdivided into Pure (or abstract) and Applied (or basic) Mathematics, they bear symbiotic relationship to one another. Abstract problems in mathematics have often proved to require, in their resolution, information from its applied counterpart or even from a section of its area. Similarly, basic studies have easily been successfully handled with the use of results from Pure or Applied Mathematics. This observation supports the notion that knowledge in the different sections of Mathematics are like a chain of interconnected rings that forms a loop.

The implication of the above observation is that the training of mathematicians should be all inclusive of fundamental areas of pure and applied mathematics, so as to equip the graduates to function effectively wherever they may settle for vocation or career.

Another attribute of Mathematics, as demonstrated by my research results is that Mathematics is applicable to many areas of human endeavours in the physical realm. Mathematics apparently and easily finds application in the sciences (natural, basic, social and medical), engineering and technology. With a contemporary world that is information-laden and technology-driven, there will hardly be any knowledge areas where Mathematics will not spead its ‘tentacles’ into.

The above observation is in tandem with the pronouncements of relevant authorities in the discipline. Carl Friedrich Gauss was the first to recognize the significance of mathematics. He referred to it as the “Queen of the Sciences”. Later, Marcus du Sautoy called mathematics "the Queen of

Science ... the main driving force behind scientific discovery". More recently, mathematics has been variously alluded to as the Queen and Servant of contemporary disciplines. It is pertinent to note that a queen is a female monarch or ruler “that is foremost or preeminent in any respect”, while a servant in the ancient human society was a privileged position bestowed on a worthy personality. Thus, Mathematics has been ascribed the dual role of a preeminent and a privileged discipline. On this premise, I make bold to assert that Mathematics is a servant to all, and a master of all!

With an emerging definition of mathematics as the study of pattern and order in abstract objects through logic, observation, simulation, and even experimentation, the discipline is enabled to study the world around us including the natural phenomena, of human behavior, and of social systems.

Galileo Galilei was quoted to have said, "The universe cannot be read until we have learned the language and become familiar with the characters in which it is written …" Roger Bacon was also quoted to have said that the things of this world cannot be made known without a knowledge of mathematics. Mathematics, perhaps, is the only subject in all education systems, from primary to tertiary, that speaks a common language, with its distinct lexis, syntax and semantics. The lexis are the notions, axioms, concepts and terminologies. The syntax includes properties, conjectures, propositions, and theorems, while its semantics are combinations of abbreviations, notations, symbols. The import here is that Mathematics is a universal discipline that evolves a language that can be read, ruminated and communicated by people from all nations on earth.

Mathematics is incredibly important in our lives. We use mathematical concepts and skills consciously or unconsciously in our daily activities. Even the natural activities abounding in our surroundings have been encoded using the laws of Mathematics. Thus, we need Mathematics in order to make our life better, orderly and prevent chaos.

It is a known fact that everyone is either a professional, user or lover of Mathematics. At the level of users are the majority, who consciously or otherwise apply mathematical principles and concepts from time to time in their business, commerce or occupational endeavors. In the minority are the people who take up a career in the discipline, and thereby contribute, directly or indirectly, to the development of Mathematics. Notwithstanding these obvious divides, to cope with contemporary demands as a result of rapid development in the field of technology, a minimum level of literacy in mathematics is advocated.

Recommendations

Mr. Vice-Chancellor, Sir, as I begin to give recommendations based on today's lecture titled: **Making the Best Use of What You Have: The Mathematical Approach**, kindly permit to briefly analyse a keyword in the title. The word "You" has a dual significance. It has reference to each one of us as individuals, and to us collectively as a group or corporate body.

Advice to Individuals

As individuals, each one of us is enjoined to make best use of the resources s/he is endowed with. In similitude of a song we used to sing in the Primary School during Hygiene lesson, an equivalent admonition concerning your life is composed as below.

"Take good care of your life;
It hasn't got duplicate
When you're gone,
You're gone forever;
You're n't coming back."

The same encouragement holds for your health.

Let me leave you with the following guidelines on how to optimize one of your valuables:

- Create a budget for your financial plan.

- Save for short term needs.
- Invest for your long term needs.
- Use credit wisely and responsibly.
- Cut your coat according to your clothes - buy things you can only afford.
- Don't be too frugal, enjoy your money while you are still alive.
- Keep learning about financial tools and resources to be secured, ARAG (2021).

Advice to Corporate Bodies and Government

Mr. Vice-Chancellor, Sir, distinguished guests, ladies and gentlemen, I am now addressing the remaining recommendations to the relevant bodies including government at all levels.

The problems militating against the realization of optimizing our national life are insecurity (terrorism, banditry, kidnapping, herders' problem etc.), unemployment, economic challenges, unstable electric power supply, just to name a few. These are taking heavy tolls on the well-being of our nation in terms of untimely deaths, fear and apprehension, hyper-inflation, poverty and low esteem in the comity of nations. The problems require scientific approach in their resolution. Such a technique should be multi-tasking since some of the highlighted problems affect other areas. As such, I am recommending to the Nigerian Government the Mathematical Optimization presented in the lecture as a way out of some of these challenges.

Techniques of Mathematical Optimization can be employed to enhance the economic fortunes of a nation. It is amazing to find how USA, Russia, China, India, South Africa grow their national development with the application of Mathematics. By controlling key parameters of our economy, fluctuations in prices can be kept abreast. I commend the Nigerian government for not increasing the pump price of petrol despite fluctuations in the prices of crude oil in the international market. However, some other sensitive parameters should be identified and controlled to curtail their influence on the economy. It is apparent that the current sky-rocketing of the cost of essential items is as a result of the instability of the dollar-naira exchange rates.

Two important strategies utilized in Mathematical Optimization are ensuring system's improvement and the elimination of wastages. Policies and practices that are inimical to progress and growth should be avoided. Democracy guarantees freedom, nonetheless it has never been a cloak for wastages. Unlike many democratic settings, our administrative setups condone a lot of wastages. Even corruption is a high-level mode of wastage of the resources of a sovereign. If we want to develop to the best of our potentials, our governments must summon more courage to fight the 'monster' called corruption to a standstill, albeit, within the ambits of the constitutional provisions.

I hereby suggest that an adhoc panel or commission be raised to identify areas of wastages and proffer ways to ameliorate them.

Nigeria seems to be neck-deep into external debts again after the debt crisis of 2005. Since, 2007 the debt profile has been on the increase. In 2019, Nigeria's government spent 59.6% of her total revenue on debt service. One of the tenets of Systems Optimization stipulates that expenditures in a system must be as low as possible before the system can attain the highest performance. A country like Nigeria, whose major source of revenue is a declining natural resource and as a nation beset with unfavourable cultural, economic and political environments, we may not find it easy to get out of another debt crisis. The legislature and the executive arms of the Nigerian government should put in place a legal framework that will ensure drastic reduction, to the minimum level, of our external debt profile.

One ominous sign of underdevelopment in our country is the incessant outages in electricity supply. Disruptions in electricity supply not only interrupts processes, service delivery, but is the harbinger of the many dangers that bedevil such a community. The Nigerian government should rally support to enable regular supply of electricity.

There should be regular revision of mathematics curricular at all levels of education for the purposes of adapting to the rapidly expanding frontiers of mathematical knowledge in concept, content, methodology and adaptation. Normally, a curriculum is adjudged to be getting obsolete after ten (10) years of its implementation. This suggestion becomes necessary in order that students may be adequately equipped for work, life and other challenges.

It is common knowledge that many students that are enrolled into Mathematics-related courses in tertiary institutions are those who cannot find admission spaces in professional science and engineering courses. I suggest the institution of a scholarship programme as a way of attracting talented students to study mathematics. This can be part of the mandate of the National Mathematical Centre at Abuja.

In recognition of the importance of mathematics to national and global development, the federal government should encourage careers in mathematical research through the provision of generous scholarships for this purpose and through making University teaching and research more lucrative.

Acknowledgements

I return all thanks and praises to God who has piloted the affairs of this day thus far. He spared my life, gave me wisdom and knowledge to reach the pinnacle of my career. He has led me to know that the future belongs to Him, and if one will follow His guidance, the prospects of our future can be optimized for eternity.

I gratefully appreciate my father, Late Pa Peter A. **Bamigbola**, who introduced me into the world of everyday optimization. I remember his frequently admonition spoken in Igbomina dialect: " … be ba wi ko ha gbe, o gbodo ra a …" translated as: "… don't buy the item until you have priced it to the point of asking you to come and steal it …" I rejoice with my mother, Princess Florence M. **Bamigbola** - the best of all mothers, for being counted worthy to witness today's occasion!

I acknowledge the **Bamigbola** family, both the immediate and extended descent. Worthy of appreciation are my sister, Mrs. Dorcas O. Omotosho, and her late husband, Estate Suryeyor S.A. Omotosho, both of whom positively impacted on my life.

I very much appreciate my other siblings: Messers Shola, Jide and Lanre **Bamigbola** & their spouses. I also appreciate Mrs. Nimota Adigun & husband, Mrs. Tawa Fadipe, Mr. Rafiu **Bamigbola** and others from the extended **Bamigbola** descent including those from Olayinka town.

I specially thank my fathers and mothers from the Olukotun Compound of Ijan-Otun notably, Sir Honourable Justice (retired) & Mrs. T.A. Oyeyipo, Chief & Mrs. N. K. Omolohunnu, Mrs. Comfort Oyinloye, Mr. & Mrs P.R. Odewale, Mrs. E. Olabanji, Mrs. D.M. Oladele & husband and Mrs. C.O Adewumi & husband.

I gratefully acknowledge his Royal Highness, the Onijan of Ijan-Otun, Oba Aaron Omoniyi Aransiola (Aragbabalo III) and his Chiefs for gracing this occasion.

I appreciate Dr. & Mrs. E.F. Awotundun, Chief & Mrs. J.B. Oyeleke, Mr. &Mrs. D.A. Olawepo and Rev. & Mrs. J.B. Ibiwoye for supports.

I express sincere appreciation to my in-laws Mr. & Mrs. S.O. Awotayo, Mr. A.O. Awotayo, Rev. & Mrs. D.O. Idowu, Mr. & Mrs. E. Jimba, Mr. & Mrs. Odeyemi, Mrs. B.M. Yusuf, and others that time will not permit me to mention.

Special acknowledgement is due my supervisor and academic mentor, late Distinguished Professor M.A. Ibiejugba, and that of my PhD co-supervisor, Professor P. Onumanyi.

I whole heartedly thank Professor J.A. Gbadeyan for many roles he has played: Academic adviser, friend and supporter.

I appreciate the cordial family-like rapport in the Department of Mathematics, University of Ilorin made possible through the solid groundworks of the founding fathers: late Professor E.A. Bangudu, Dr. E.O. Ogunsulire, and Professors M.A. Ibiejugba, P. Onumanyi and J.A. Gbadeyan as well as the faithful followership of the present crop of colleagues: Professors K. Rauf (Head of Department), T.O. Opoola, M.O. Ibrahim, O.A. Taiwo, R.B. Adeniyi, A.S. Idowu, K.O. Babalola and M.S. Dada, as well as Doctors E.O. Titiloye, Olubunmi A. Fadipe-Joseph, Yidiat O. Aderinto, Catherine N. Ejieji, B.M. Yisa, J.U. Abubakar, K.A. Bello, Gatta N. Bakare, B.M. Ahmed, O.A. Uwaheren, Idayat F. Usamot, O.T. Olotu, O. Odekunle, A.Y. Ayinla, T.L. Oyekunle. The various non-teaching staff who shared in the camaraderie are also acknowledged.

I appreciate my academic friends: Professors K. Oje, K.L. Ayorinde, B.L. Adeleke and M.O. Adewumi.

I highly appreciate the input of the Library and Publications Committee under the leadership of Professor A.K. Salami.

I gratefully acknowledge my collaborators especially Professor Montaz M. Ali of University of Witwatersrand, Johannesburg, South Africa, late Professor Aderemi Adewumi of KwaZulu Natal University, Durban, South Africa, late Professor Timothy A. Adewale of Adekunle Ajasin University, Akungba Akoko, Nigeria and Dr. (Engr.) Kehinde Awodele of the University of Cape Town, South Africa.

I respectfully thank the past and present Vice-Chancellors and Management teams of the University of Ilorin for giving me the unique opportunity to study to earn the three academic degrees, and work as a lecturer and researcher in this academic citadel. My special thanks to Professors A. Adeniyi (late), S.O. Amali and S.A. Abdulkareem during whose respective tenures I was employed as an Assistant Lecturer, promoted as a Professor of Mathematics and presented my Inaugural Lecture.

I sincerely appreciate the leaders, ministers, other officials and the body of saints of the Apostolic Faith Church, among whom I have been receiving the heavenly nourishment

which has sustained my spiritual life since 1973. I extend my special appreciation to the past and present heads of the church in West and Central African District. I gratefully acknowledge Rev. J.O. Ajiboye (late), whom God used to bring me to the fold, Rev. F.A. Akinboye (late), who counselled me to take to the teaching profession, Rev. P.K. Shidali (foundation pastor at Ilorin) and Rev. F.S. Fadipe (present Kwara District Overseer) for spiritual support.

It was like yesterday, but I thank God for standing by me when my lovely, when the cold hand of death snatched my soul-mate and wife, Mrs. Rebecca Olubunmi Bamigbola to the Great Beyond on 16th April, 1996. In keeping to His promise in Genesis 2 verse 18, God provided a beautiful, lovely and caring "help meet" in person of Dr. Esther Olayinka **Bamigbola** (nee Jimba) who through God's help has been a wonderful support at all fronts. Darling, thank you very much.

I thank God for the wonderful, lovely and God-fearing children He has given us. Mary Oluwakemi Oluwayomi, Christianah Tolulope, Rejoice Oluwatosin, Joy Oluwafemi, Love Oluwaseun, Gift Oluwabusayo and Gem Oluwafeyisayo, I am proud of you all. I extend appreciation to our in-law Mr. Sunday Oluwayomi. To you my granddaughter, Deborah Joan Oluwayomi, God bless you real good.

I specially thank you all, whether physically or virtually present, for honouring my invitation to attend this Inaugural Lecture. I pray God bless you richly, Amen.

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